Bubble Creation and Multi-fluids Interaction

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Abstract

In this article, we propose a physically based novel scheme on the bubble creation. By the scheme, the phase change of the liquid-gas interaction and the effect of volume expansion are simulated by altering the divergence of the velocity field. Because of the little mass of the gas and the great difference in density between liquid and vapor, the volume of the liquid is considered to be constant. We construct the multi-material interface using the volume of fluid (VOF) and piecewise linear interface construction (PLIC) methods and maintain the volume conservation for each material.

1. Introduction

The key problem of the immiscible fluid simulation is how to describe the fluid interface and the motion of the fluid interface. The common method for the description of the fluid interface is by tracking [1], [2] or capturing of the interface [3]–[8]. By the tracking method, the interface position is recorded by a mesh at the initial time, and then, the vertices of the mesh are moved in the space like Lagrange particles, according to the calculated velocity field. The tracking method can be used to exactly describe the position of the interface and calculate the surface tension, but when the interface is breaking or merging, it would be difficult to handle the geometric change of the interface mesh, so that the result becomes not exact. By the capturing method, the interface position is recorded in terms of a scalar field, and while the interface is moving, the scalar field is updated according to the fluid flux, or through the massless Lagrange particles. This method can be easily used to describe the interface breaking or interface merging. However, by the discrete sampling of the space, there exists a critical frequency according to the sampling theory, so the scalar field will be unable to precisely describe the model part in great detail with high frequency. Therefore, because the scalar field cannot correctly describe the interface position, we cannot get the curvature and calculate the surface tension exactly at the same time. According to the meaning of the scalar field, the interface capturing method can be divided into Level Set method [3], [7] and the VOF method [4]–[6], [8]. The Level Set method records the minimum distance to the interface for each space point. It can easily get the smooth and continual interface, but it is difficult to maintain the volume under the interface during the simulation. The VOF records the volume fraction of the fluid in each computation cell. It can easily maintain the volume under the interface, but, it is difficult to get a smooth and continual interface.

In our life, we can see various applications related to the phenomena of bubble, such as boiling, chemical reaction, and variety of papers were published on the bubble simulation in the physical and computer graphic field [9]–[11]. To solve the problem of the bubble creation, a common method is by solving the equation of state [12], or taking the vapor as a compressible fluid [13]. The method with the equation of state is to calculate the pressure of the vapor according to the information of vapor energy, vapor entropy, vapor volume, vapor temperature, and then apply the vapor pressure as the boundary condition for the liquid to conduct the vapor-liquid coupling. But when an underwater explosion occurs, the resultant energy release creates an expanding vapor bubble which undergoes a multiple expansion (explosion) and collapse (implosion) process and continuously loses energy until it breaks down. Nevertheless, the bubble oscillation process, the initiation process, the source of the instability leading to bubble collapse, and the energy loss mechanism are still not completely understood [14]. Therefore, it is difficult to make precise analysis to the whole process. The method of considering the vapor as a compressible fluid is to use the compressible fluid motion equation to simulate the vapor motion. Because the vapor expansion is very fast, there will be shock in the fluid and the shock will reflect between the interface of the vapor and the liquid. The whole process is very complex and there is still no good way so far to capture the shock exactly. Our method is to use a simplified process to simulate the bubble expansion. We take the vapor as an incompressible fluid and introduce the volume expansion, according to the V.K.Dhirs work [15], into the divergence of the velocity field to simulate the liquid-gas phase change and the vapor expansion.
In the computer graphic and the physical world, much work has been done on the subject of free-surface flow [16] and the two-material fluid simulation [17]–[19]. However, there is still no good way to deal with the multi-material ($M \geq 3$) flow. Losasso et al. [7] give a solution for the Level Set method and use the average distance of all the material to solve the overlap of the fluid regions. However, the volume of each material cannot be maintained in their solution. For the VOF method [20]–[22], the key problem is how to determine the sequence of calculating the interface and how to calculate the normal for each interface. Here, we use the sequence selected by the user before the simulation and use the onion-skin scheme to calculate the normal of the interface.

2. Algorithm

2.1. Bubble creation

We use the Eulerian formulation of the equations to describe the inviscid incompressible fluid [23]. The principal motion of the flow is governed by the basic conservation laws for the momenta and mass.

$$u_t + (u \cdot \nabla)u = -\frac{1}{\rho} \nabla p + g + \frac{1}{\rho} S$$

(1)

$$\nabla \cdot u = 0$$

(2)

Here, $u$ is the fluid velocity, $p$ is the pressure of the fluid, $g$ is the gravity and $S$ is the surface tension. Equation (1) governs the momenta conversation, and equation (2) indicates the mass and volume conservation. The common method to solve these equations is the projector-corrector method.

When the bubble is expanding, part of the liquid is changed into the vapor at the interface of the bubble. So the mass of the liquid decreases and the mass of the vapor increases, but the total mass of the liquid and gas should be maintained. According to the work of V.K. Dhir [15], to keep conservation of the whole mass, the condition of the mass continuity at the interface must be maintained. So we get

$$m = \rho_{vapor}(u_i - u_{vapor}) = \rho_{liquid}(u_i - u_{liquid})$$

(3)

$u_i$ is the interface velocity. Arranging the left and right side of equation (3) to get ride of $u_i$, we can get
When the bubble starts to float upward, we change because of the small volume of the bubble and the fast reaches 2–4 times in the size of the cell. At the same time, and keep the volume constant after the radius of the bubble created. So, we suppose that the bubble will float upward from the bottom soon after it is created. The demo shows the whole process of the bubble creation. During the expansion time, the volume of the liquid is maintained.

2.2. Multi-fluids Interaction

VOF [4]–[6], [8] and PLIC [4], [24] method are originally used for the free-surface problem. VOF records the volume ratio of the fluid to the cell; PLIC presumes that the interface segment in the cell is a plane. In the process of the interface reconstruction, first of all, we calculate the normal direction, according to the height function of the VOF; and then, decide the interface position according to the volume fraction and the normal direction.

For the multi-material interface reconstruction, there are two different schemes [20]–[22] in the physical field and we write here for completeness. Assuming that there are M materials:

**Scheme 1:** In a completely independent manner calculate M linear interfaces inside the mixed cell, such that the m-th interface, \( m \in [1, M] \), separates the m-th material from the rest. The m-th material occupies the space under the m-th interface, \( m \in [1, M] \).

This algorithm is always 1st-order accurate.

**Scheme 2 (onion-skin):** Given a particular material order, in a completely independent manner calculate M-1 linear interfaces inside the mixed cell, such that the m-th interface, \( m \in [1, M] \), separates the mixture of the first m materials from the rest.

Under the assumptions that:

1) The true partition is C2-parallel (i.e. the interfaces do not intersect and are C2-differentiable).
2) The right material order is given.
3) The (two-material) interface reconstruction algorithm used is 2nd-order accurate.

Then, the algorithm is 2nd-order accurate.

Here, we use the Scheme 2 for both the interface reconstruction and interface advection. The material order is given as the material sequence. For some mixed cell, the given order may be not correct, and the algorithm is the 1st-order. But, the result shows that the overlap regions of the fluid are small and the result is acceptable.

\[
\rho_{\text{liquid}} - \rho_{\text{vapor}} = (\rho_{\text{vapor}}^{-1} - \rho_{\text{liquid}}^{-1})m
\]  

Equation (4) expresses the velocity divergence near the interface. And, we introduce the step function \( H \), which is zero at the vapor side and one at the liquid side. Then, the equation (2) can be written as

\[
\nabla \cdot u = (\rho_{\text{vapor}}^{-1} - \rho_{\text{liquid}}^{-1})m \cdot \nabla H
\]

Finally, the control equation for the fluid is

\[
u + (u \cdot \nabla)u = \frac{1}{\rho} \nabla p + g + \frac{1}{\rho} S
\]

\[
\nabla \cdot u = (\rho_{\text{vapor}}^{-1} - \rho_{\text{liquid}}^{-1})m \cdot \nabla H
\]

The problem can be simplified further. We assume that, at the unit time, there is constant mass change from the liquid to the vapor, so the volume expansion of the bubble is constant. And, the equation (7) can be written as

\[
\nabla \cdot u = \Delta V
\]

And the constant mass change can be written as

\[
\frac{\Delta V}{\Delta t} = \Delta C \cdot V_{\text{cell}}
\]

Then, combine the equation (8) with the equation (9), and get

\[
\nabla \cdot u = \Delta C \cdot V_{\text{cell}} \cdot \Delta t
\]

The volume of the vapor can be updated as

\[
C_{\text{vapor}} = C_{\text{old}} + \Delta C
\]

Here, C is the fraction of the vapor in the cell, \( V_{\text{cell}} \) is the volume of the cell, and \( \Delta t \) is the computation time step.

In reality, because of the effect of the surface tension, the bubble will float upward from the bottom soon after it is created. So, we suppose that the bubble will float upward and keep the volume constant after the radius of the bubble reaches 2–4 times in the size of the cell. At the same time, because of the small volume of the bubble and the fast expansion, the geometry shape of the bubble is unchanged during the bubble expansion and the shape is the perfect sphere. When the bubble starts to float upward, we change the VOF value in each cell according to the position of the bubble interface and then use the common projector-corrector method to simulate the bubble.

In the Fig (1), we show how the bubble is created. At the initial time, there is no bubble in the liquid. Then, the bubble is created at the bottom of the liquid. And then, the bubble is expanding at the bottom of the liquid. Because the bubble is attached to the bottom, the bubble only increases the volume and doesn’t float upward. At last, the bubble is big enough. And because of the surface tension, the bubble is separated from the bottom and begins to float upward.

Here, we use the Scheme 2 for both the interface reconstruction and interface advection. The material order is given as the material sequence. For some mixed cell, the given order may be not correct, and the algorithm is the 1st-order. But, the result shows that the overlap regions of the fluid are small and the result is acceptable.
Figure 2. (a) Bubble floating upward in the first material; (b) Bubble floating upward through the interface; (c) Bubble floating upward through the free surface; (d) bubble breaking up.

Fig (2) shows the scene, in which the bubble is floating upward through the water and the oil. At the initial time, the bubble is at the region of the water. Then, the bubble floats upwards through the water and the oil. At last, the bubble breaks up at the free surface.

3. Conclusion

We propose a physically based novel scheme on the bubble creation. By the scheme, the phase change of the liquid-gas interaction and the effect of volume expansion are simulated by altering the divergence of the velocity field. And the VOF and PLIC methods are applied, combining with the onion-skin scheme, to handle the multi-fluid interaction.

In the current multi-fluid scheme, we only deal with the immiscible fluids, and do not include the surface tension. The system is sensitive to the noise of surface tension, which makes the system unstable. In the future work, we will use large viscous force and the smoothed surface tension to make system stable, and add the diffuse effect to put the immiscible and miscible fluid simulation in the same framework.

In the demo, the volume of the bubble changes when it floats up. It is caused by the coarse computation grid. So, in the future work, we will use the adaptive grid to improve the effect.

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