Dynamic Behavior of SIS Epidemic Model with Feedback on Regular Lattice

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Abstract—In this paper, the dynamic behavior of SIS epidemic model with feedback mechanism on regular lattice is discussed. We investigate the effects of feedback mechanism, crowd density, spread efficiency and the moving activity of individuals on disease spreading by theoretical analysis and computer simulation. The results indicate that feedback mechanism can impact the stable infective ratio of system greatly, moreover, the effect of feedback mechanism in the case of moving crowd is better than that of static crowd, and however, it can not impact the spreading threshold of system. In addition, the results also show that the moving activity of individuals can promote disease spreading under the condition of low crowd density.

Keywords—SIS epidemic model; feedback mechanism; crowd density; regular lattice

I. INTRODUCTION

The spreading of computer virus, the epidemic of disease and the promulgating of rumor can all be considered as a dynamic behavior which obeys similar laws on networks. It is of great practical significance to research and understand the laws of such dynamic behavior on networks. The theory of complex networks provides us a new way to understand the dynamic behavior, reveal its characteristics and search for effective control methods.

People have researched the spreading behavior of disease for a long time. The traditional research method is to assume that individuals in networks are in several typical states respectively. The basic states include susceptible state (S); infective state (I) and recovered state (R). The conversion process among these states is usually used to name different model. The classical models are susceptible-infective-susceptible (SIS) and susceptible-infective-recovered (SIR). Base on these models, people find that there is a positive critical threshold of spread efficiency on homogeneous networks (such as regular networks and small-world networks). If spread efficiency is below the threshold, the disease will rapidly disappear exponentially, that is to say, the disease will not spread on networks. Only spread efficiency is above the threshold, can the large scale spreading of disease become possible. However, the threshold tends to zero on heterogeneous networks (such as scale-free networks). This means that even a little source of infection can arouse a large scale spreading of disease on heterogeneous networks. Although these models indicate some laws of disease spreading, there are still some factors being ignored. These factors usually have a significant impact on the process of disease spreading. In this paper, there are three additional factors (feedback mechanism, crowd density and the moving activity of individuals) to be considered in classical SIS model. Firstly, we build theoretical model. Secondly, we research the effects of the three factors by computer simulation. In the end, we analyze the results of simulation in detail.

II. THEORY MODEL

A. Model Building

Put crowd into L×L regular lattices sparsely and each lattice can only accommodate one individual. Assume the ratio of total number of individuals to the total number of lattices is denoted by crowd density \( d \). At the initial moment, there are a few infected individuals in the networks. At each time step, every infective individual can infect its four neighbor individuals who are upper, lower, left and right with rate \( \alpha \), meanwhile, the infective individual can be cured with rate \( \beta \). At the \( t \) moment, to assume the ratio of susceptible individuals is \( s(t) \) and the ratio of infective individuals is \( i(t) \). According to idea of [12], we suppose that the individuals can know the development of disease spreading in the networks at every moment, that is to say, the infective ratio at the last time step can be known, then the individuals can disconnect with the infective individuals voluntarily with rate \( \delta \), so the rate that every individual disconnects with its neighbors is \( \delta(i(t-1)) \). \( \delta \) is called feedback measure parameter. The value of \( \delta \) is between 0 and 1. Obviously, \( \delta \) is related to the severity of the disease, if the disease is serious, the value of \( \delta \) is high, otherwise, it is small. When time \( t \) tends to infinity, \( i(t) \) is a stable value, so \( i(t)=i(t-1) \). According to the mean-field theory, the process can be described by equations below:

\[
\begin{align*}
\frac{ds}{dt} &= -\alpha<k>si + \beta i \\
\frac{di}{dt} &= \alpha<k>(1-\delta)i - \beta i
\end{align*}
\]

(1)

Where, \( <k> \) is the average number of susceptible individuals around an infected individual.

Because \( s(t) \) and \( i(t) \) are linked through the normalization condition: \( s(t)+i(t)=1 \), (1) can be simplified as:
\[ \frac{d}{dt} = \alpha(k)(1-\delta)i(1-i)i-\beta i. \]  

(2)

Because of \((1-\delta)i(1-i)=i-i^2-\delta^2+\delta\delta'=1-(1+\delta)i\), (2) can be simplified as

\[ \frac{d}{dt} = \alpha(k)[(1-(1+\delta)i)i-\beta i. \]  

(3)

If the system is in stable state, then \( \frac{d}{dt} = 0 \), so

\[ I = \begin{cases} 
0 & \alpha(k) < \beta \\
[1-\frac{\beta}{\alpha(k)}][1+\delta)]=\frac{1}{4} & \alpha(k) > \beta.
\end{cases} \]  

(4)

For the sparsely distributed regular lattices, we can consider \(<k>\) is related to crowd density and can be denoted as \(<k>=4d\). Put it into (4), we can get

\[ I = \begin{cases} 
0 & (\lambda d < \frac{1}{4}) \\
[1-\frac{\beta}{\alpha(k)}][1+\delta)]=\frac{1}{4} & (\lambda d > \frac{1}{4}).
\end{cases} \]  

(5)

Where, \( \lambda=\alpha/\beta \) is defined spread efficiency.

Obviously, the stable infective ratio is not only related to \( \lambda \) and \( d \), but also related to \( \delta \). The bigger the value of \( \delta \) is, the smaller the stable infective ratio \( I \) is. In addition, the critical threshold of disease spreading is determined by both \( \lambda \) and \( d \).

If \( \lambda \) is fixed, there is a crowd density threshold \( d_c \), meanwhile, if \( d \) is fixed, there is a spread efficiency threshold \( \lambda_c \).

B. Stability Judgment

If \( I=0 \), the Jacobi matrix of (3) is \( J = \alpha(k)-\beta \), furthermore, the eigenvalue of the matrix is \( \alpha(k)-\beta \). For \( \alpha(k)<\beta \), so \( \alpha(k)-\beta<0 \). According to the stability criterion of nonlinear equations' solution, \( I=0 \) is a local stable solution of (3) under the condition of \( \alpha(k)<\beta \).

If \( I = [1-\frac{\beta}{\alpha(k)}][1+\delta)]=\frac{1}{4} \), the Jacobi matrix of (3) is \( J = \beta-\alpha(k) \), furthermore, the eigenvalue of the matrix is \( \beta-\alpha(k) \). For \( \alpha(k)>\beta \), so \( \beta-\alpha(k)<0 \). That is to say, \( I = [1-\frac{\beta}{\alpha(k)}][1+\delta)]=\frac{1}{4} \) is also a local stable solution of (3) under the condition of \( \alpha(k)>\beta \).

III. COMPUTER SIMULATION

In order to test the results of above model, we do simulation experiments on two-dimension regular lattices. The simulation will be carried out in the case of static individuals and moving individuals respectively.
Then to set $d=0.5$ fixed, $\beta=0.2$, $\alpha$ is variable, so $\lambda$ is variable. The relationship between $I$ and $\lambda$ is shown in Fig.4.

It is shown in Fig.3 and Fig.4 that $\delta$ has no influence on the crowd density threshold $d_c$ and the spread efficiency threshold $\lambda_c$.

From Fig.3, we can know that when $\lambda=1$ is fixed, the crowd density threshold is $d_c=0.3$, that is to say, only crowd density is above 0.3, can the disease spread continuously and steadily. It is shown in Fig.4 that the spread efficiency threshold is $\lambda_c=0.6$ when $d=0.5$ is fixed, only spread efficiency is above 0.6, can the disease spread continuously and steadily.

B. Moving Crowd Simulation

In this simulation, we investigate the impact of feedback mechanism on the disease spreading in the case of moving crowd. The moving process is devised as follows: At each time step, an infective individual will select a location randomly in the lattices, if the target location has been occupied by other individual, to exchange the positions between them.

The simulation parameters are the same as those of static crowd. The simulation results are shown in Fig.5, Fig.6, Fig.7 and Fig.8.
It is shown in Fig.5 and Fig.6 that feedback mechanism is also effective in the case of moving crowd. To compare Fig.1, Fig.2 with Fig.5, Fig.6, we can know that if δ is the same value, the effect of feedback mechanism is more obvious when the crowd is moving. In addition, we can know that when d is high (for example: d=0.8), the infective ratio of moving crowd is nearly the same as that of static crowd, however, when d is low (for example: d=0.5), the infective ratio of moving crowd is obviously higher than that of static crowd. This indicates that crowd moving is good for the disease spreading under the condition of low crowd density.

It is shown in Fig.7 and Fig.8 that feedback measure has no impact on the crowd density threshold δc and the spread efficiency threshold λc in the case of moving crowd.

IV. CONCLUSION

In this paper, the dynamic behavior of SIS epidemic model with feedback mechanism on regular lattice is discussed in detail. We investigate the effects of feedback parameter, the crowd density, spread efficiency and the moving activity of individuals on disease spreading. The results indicate that feedback can impact the stable infective ratio greatly, the bigger the parameter δ is, the smaller the stable infective ratio I is, however, it can not impact the spreading threshold of the system. Moreover, the effect of feedback mechanism in the case of moving crowd is better than that of static crowd. In our understanding, in the case of static crowd, the networks will show the phenomenon that the infective individuals accumulate in local area. The feedback mechanism will lose its function in the local regions. The moving activity of individuals can reduce the effect that the infective individuals accumulate in local region, so the function of feedback mechanism is better in the case of moving crowd. In addition, the results also show that the stable infective ratio of moving crowd is obviously higher than that of static crowd under the condition of low crowd density. In our understanding, under the condition of low crowd density, if the crowd is static, there are some isolated individuals in networks, they can not be infected if they are susceptible, meanwhile, they can not infect other individuals if they are infective. That is to say, the isolated individuals in networks can hold down disease spreading. The moving activity of individuals can reduce the number of isolated individuals, so the stable infective ratio of moving crowd is higher. However, if crowd density is very high, there are few isolated individuals in networks in the case of static crowd, so the stable infective ratio of static crowd is nearly the same as that of moving crowd.

According to the conclusions above, in order to control disease spreading, we can adopt the prophylactic and controlling schemes as follows:

(i) Collecting and promulgating epidemic information in time to let people disconnect with the infective crowd voluntarily.
(ii) Reducing large-scale crowd meeting to avoid high crowd density in local district.
(iii) Restricting the frequent moving of massive crowd to reduce the disease spreading effectively.

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