Cramer-Rao lower bound for non-coherent TOA estimation with Impulse Signal

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Abstract—The non-coherent time-of-arrival (TOA) estimations are applied in impulse location for its simplicity. However, the multipath propagation of the narrow pulse results in complicated performance analysis. By fitting the output envelope of energy detection into a exponential decay, we approximately deduce the closed-form Cramer-Rao low bound (CRLB) of TOA estimation. Simulations with non-coherent TOA algorithms, such as threshold crossing (TC) and maximum energy selection (MES), validate the analysis. Furthermore, the integration window and multipath delay spread in ultra-wideband channel are considered to obtain a referable bound.

I. INTRODUCTION

UWB ranging based on TOA will be an important role in wireless communication in future. Due to the low complexity, TOA algorithms based on non-coherent detection are applied more and more frequently on UWB ranging. Cramer-Rao low bound (CRLB) analysis can effectively evaluate performance of distance measurement.

In ref. [1], CRLB is deduced to evaluate the performance of coherent detection based TOA estimation. However, the likelihood function used in the deduction cannot be directly applied for non-coherent case. Some non-coherent TOA estimations are introduced to impulse ranging for their low-complexity implement. For instance, a two-step non-coherent TOA algorithm is proposed to obtain a highly precise measurement [2], whereas it have no theoretical analysis for energy detection. With the joint probability density functions of path gain and the desired time delay, CRLB is given in ref. [3] assumed a prior known channel, Caticov presents a TOA/RSS scheme and the CRLB analysis is also based on coherent TOA estimation [4]. In this article, emphasis is how to deduce closed-form of CRLB for energy-detection based on TOA. With the obtained closed-form of CRLB, integration and RMS delay influence on CRLB based on non-coherent is analyzed.

II. SYSTEM MODEL

Let the received UWB impulse signal be presented as

\[ r(t) = \sum_{i=0}^{\infty} d_i P_{mp}(t-jT_c+cT_i) + n(t) \]  \hspace{1cm} (1)

where pulse duration is \( T_p \), \( d_i \) and \( T_c \) are the fading coefficients and delays of the multipath components, respectively. Especially, \( T_c \) is DP (direct path). \( E_s \) is the symbol energy, \( N_i \) represents the number of pulses per symbol. We assume that \( T_i > T_{\text{ch}} + c_{\text{max}}, T_c \) to avoid Inter-symbol interference, where \( T_{\text{ch}} \) is the effective duration of channel impulse response, and \( c_{\text{max}} \) is maximum value of time-hopping code.

In non-coherent energy detection, output from square-law device, the received impulses are integrated and sampled with interval \( T_s \). The number of energy samples is \( N_s = \lceil T_s / T_c \rceil \). To obtain reliable result, we combine the square-law impulses from several frames to reduce the variance of the noise item. That is, \( Y_n = \sum_{i=0}^{N_s} Y_{n,j} \), where \( Y_{n,j} \) is the sample value of the squared impulse, \( Y_{n,j} \) denotes the sample index from the starting point of the uncertainty region. \( Y_{n,j} \) is denoted by

\[ Y_{n,j} = \int_{(j-1)T_s}^{(j-1)T_s + nT_c} |r(t)|^2 dt \]  \hspace{1cm} (3)

where frame duration and frame index are respectively denoted by \( T_f \) and \( j \), \( T_f \) is the chip duration, the number of chips per frame is \( N_f = T_f / T_c \) and the time hopping codes \( c_j \in \{0,1,\ldots,N_c - 1\} \). Additive white Gaussian noise (AWGN) is normal distribution with zero-mean and double-sided power spectral density \( N_0/2 \) and variance \( \sigma^2 \), \( d_i \in \{\pm 1\} \) is random polarity serial. Single pulse of channel impulse response is given by

\[ p_{mp}(t) = \sqrt{E_s / N_s} \sum_{i=1}^{K} a_i p(t - \tau_i) \]  \hspace{1cm} (2)

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Fig.1 Block diagram for non-coherent energy detection

III. CRLB ANALYSIS BASED ON NON-COHERENT DETECTION

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According to the common comprehension of ultra-wideband channel, the envelope of the multipath-expended
impulses can be fitted into an exponential decay curve. By
energy detection, the output waveform further approaches to
the curve due to the low-pass function. Therefore, we can
make use of ML rule to deduce the closed-form of CRLB
based on non-coherent energy detection.

Firstly, we fit the output of energy detection into
exponential decay curve as below
\[ s'(t) = A \cdot e^{-Bt+C}/D \]
where \( A, B, C, D \) are parameters for the fitted curve. To
simplify analysis, the signal is normalized as
\[ s(t) = \frac{s'(t)}{\int_0^T s'(t)^2 dt} = A \cdot e^{-Bt+(C/D)} \]

Secondly, making use of the fitted exponential curve, we can
deduce the CRLB of non-coherent with ML. The waveform
from energy detector is sampled with interval \( T_s \). The
samples are given by
\[ E_i(T_i) = \sum_{n=1}^{N_s} \int_{i-1}^{i} s(t) dt, \quad N = T_s / T_b \]
In addition, the output of energy detector can be expressed as
\[ E_i(T_i) = E_s(T_b) + \int_{i-1}^{i} \left[ n^2(T_b) + 2s(t) \cdot n(t) \right] dt \]

Define the noise item \( N_m(T_b) = \int_{i-1}^{i} \left[ n^2(T_b) + 2s(t) \cdot n(t) \right] dt \).
It obeys \( \chi^2 \) distributions, which can be approximated as
Gaussian distributions to simplify analyze [7]. The mean and
variance of \( N_m(t) \) are \( \mu_m = T_b \cdot B_w \cdot N_o \) and
\( \sigma^2_m = T_b \cdot B_w N_o^2 + 2E_b N_o \) respectively, in which \( B_w \) is
bandwidth, and \( E_b \) is signal energy per frame. The formula
(7) is simplified as
\[ E_i(T_i) = E_s(T_b) + N_m(T_b) \]
Added signal transmitted delay \( \tau \), formula (8) becomes
\[ E_i(T_i) = E_s(T_b - \tau) + N_m(T_b) \]
Thus, the likelihood function about \( E_i(T_i) \) is given by
\[ f[E_i(T_b)|\tau] = F \exp \left\{ \frac{1}{N_o} \int_0^\infty [E_i(T_b) - E_s(T_b - \tau)]^2 dt_b \right\} \]
where \( F = \lim_{M \to \infty} \left( \frac{1}{\pi N_o} \right)^{M/2} \). It can be derived that
\[ \frac{\partial \ln f[E_i(T_b)|\tau]}{\partial \tau} = \frac{2}{N_o} \int_0^\infty [E_i(T_b) - E_s(T_b - \tau)] \frac{\partial E_s(T_b - \tau)}{\partial \tau} dt_b = \frac{2}{N_o} \int_0^\infty N_m(T_b) \frac{\partial E_s(T_b - \tau)}{\partial \tau} dt_b \]

Thus, we obtain
\[ \frac{\partial \ln f[E_i(T_b)|\tau]}{\partial \tau} = \frac{4}{N_o^2} E \left\{ \int_0^\infty N_m(T_b) \frac{\partial E_s(T_b - \tau)}{\partial \tau} dT_b \right\} \]

Defining \( SN(T_b) = \frac{4}{N_o^2} \left[ (T_bB_wN_o)^2 + T_bB_wN_o^2 + 2E_bN_o \right] \),
and due to
\[ \frac{\partial E(T_b)}{\partial T_b} = \frac{\partial}{\partial T_b} \sum_{i=1}^{N_s} \int_{i-1}^{i} s(t) dt = D \cdot \sum_{i=1}^{N_s} s(T_b - i) \left( 1 - e^{-B_T/D} \right) + e^{-B_T/D} \]
we achieve
\[ SN(T_b)^2 \sum_{i=1}^{N_s} s(T_b - i) \left( 1 - e^{-B_T/D} \right) + e^{-B_T/D} \]
Defining \( R(T_b,i) = s(T_b - i) \left( 1 - e^{-B_T/D} \right) + e^{-B_T/D} \)
(4) can be derived as
\[ SN(T_b)^2 \sum_{i=1}^{N_s} s(T_b - i) \left( 1 - e^{-B_T/D} \right) + e^{-B_T/D} \]

Finally, the closed-form of CRLB based on energy
detection is deduced as

\[ \left[ \frac{\partial \ln f[E_i(T_b)|\tau]}{\partial \tau} \right] \]
Two non-coherent TOA algorithms, TC and MES algorithm, are simulated to validate the CRLB with IEEE.802.15.4a CM3 and CM4, respectively. TC algorithm is the threshold-based TOA estimation, and MES algorithm is maximum-energy-selection TOA estimation [5].

The simulating results are shown in Fig.2 and Fig.3 under LOS and NLOS environments. In the two figures, absolute transmitted delay errors from TC and MES are larger than, and approach to the deduced CRLB, at the same time.

In recent years, UWB ranging mainly focus on indoor short distance. It is not be noticed for longer distance. As it known to all, the multipath components of a impulse spread a lot and Signal-to-Noise (SNR) decrease when transmitted distance become longer. Because the exponential decay curve contains information of multipath delay spread in formula (5), hereby, we can analyze RMS delay spread how to influence on CRLB.

Parameter \( t \) in \( s(t) \) responses RMS delay spread. To approximate RMS delay spread, we truncate the envelope waveform \( s(t) \) to include 99% energy in PDF (power delay profile) . Thus, it has

\[
\int_{0}^{T_{rms}} A \cdot e^{(-Bt+C)/D} \, dt = 0.99 \times \int_{0}^{T_f} A \cdot e^{(-Bt+C)/D} \, dt
\]

\[
\Rightarrow \tau_{rms} = \frac{D}{B} \ln \left( 0.01 + 0.99 \times e^{\frac{B \cdot T_f}{D}} \right)
\]

IV. SIMULATION RESULT

TC algorithm is simulated to analyze the influence by integration window. Simulation parameters are set to that signal bandwidth is 2GHz, \( T_f = 200ns \), IEEE.802.15.4a-CM3 environment, normalized threshold of TC is 0.4. The relationship between integration window and TOA performance is obtained. In fig.4, when integration window become larger, the estimation error from TC simulation and deduced CRLB increase.

According to fig.4, we can see how to set integration window to obtain good TOA estimate performance. Moreover, CRLB play down inconspicuously when integration window falls until certain region. In fig.5, NLOS environment, going with multipath delay spread becomes longer than LOS condition, deteriorates the performance of TOA estimation.

Due to simulation in CM3 environment, the parameters \( A, B, C, D \) in (5) are set according to the length of RMS delay spread in CM3 environment. The integration windows are set to 2ns,4ns and 8ns. B is set to 3.04733, 3.54733, 4.0473, 4.54733, and 5.04733, and D is set to 73, 72.5, 72, 71.52, and 70 to fit the envelope. In a 4a-CM3 environment, with 72.425ns RMS delay spread, different error results are obtained in different integration by TC algorithm in fig. 5.

V. CONCLUSION

In the paper, the CRLB based on UWB non-coherent energy detection is deduced. According to the closed-form of deduced CRLB, the integration window and RMS delay spread influence the performance of UWB ranging. From the results, it is easy to get a tradeoff between TOA algorithms performance and complexity in design ranging system.
REFERENCES


